## Algebra 2

$2^{\text {nd }}$ Semester Final Exam REVIEW
Packet


## Part I- Rational Exponents and Radical Functions (Topic 5)

1. Simplify:
A. $\sqrt{50 x^{2} y^{5}}$
B. $\sqrt[3]{64 x^{6} y^{5} z^{4}}$

$$
5 x y^{2} \sqrt{2 y}
$$

$$
4 x^{2} y z \sqrt[3]{y^{2} z}
$$

2. Let $f(x)=12+x^{2}$ and $g(x)=5-x^{2}$. Find $g(x) \cdot f(x)$.

$$
-x^{4}-7 x^{2}+60
$$

3. If $f(x)=2 x^{2}+1$ and $g(x)=3 x$, what is $f \circ g$ ? What is $g \circ f$ ?

$$
f(g(x))=18 x^{2}+1 \quad g(f(x))=6 x^{2}+3
$$

4. What is the equation for the inverse of the function $f(x)=3 x-4$ ?

$$
f^{-1}(x)=\frac{x+4}{3} \text { or } \frac{1}{3} x+\frac{4}{3}
$$

5. Find the inverse for the function $f(x)=-4 x^{7}$.

$$
f^{-1}(x)=7 \sqrt{\frac{-x}{4}}
$$

6. Is the inverse relation a function?

yes

no

yes
7. Graph the given function and state the family of functions to which it belongs.
a. $f(x)=\sqrt{x+2}-4$
b. $f(x)=-\sqrt[3]{x-3}$


Domain: $x \geq-2$ Range: $y \geq-4$
8. Write the equation of the function that is obtained by reflecting the function $f(x)=\sqrt{x}$ over the $x$-axis, shifting it right 6 units and up 3 units.

$$
g(x)=-\sqrt{x-6}+3
$$

9. Describe how to obtain the graph $f(x)=\sqrt[3]{x+6}-2$ from the graph of $f(x)=\sqrt[3]{x}$.

$$
6 \text { left, down } 2
$$

10. What is the domain and range?
a)

$D: x \geq 0$
A : $y \geq 1$
Solve the equation
b)


$$
\begin{aligned}
& D: x \geq 1 \\
& R: y \geq 0
\end{aligned}
$$

c) $f(x)=\sqrt{x+2}-4$
$D: \quad x \geq-2$
$R: \quad y \geq-4$
13. $\sqrt{12 x+1}=x+3$.

$$
x=\frac{13}{3}
$$

12. $\sqrt{x+12}=x$
$x=4$
$x=4 \quad x=2$

14 The population of the United States in 1994 was about 260 million with an average annual rate of increase of about $0.7 \%$.
a. Find the growth factor for that year. 1.7
$\begin{array}{ll}\text { b. Suppose the rate of growth continued to be } 0.7 \% . & y=260(1.7)^{t} \\ \text { Write a function to model this population growth. }\end{array}$ Write a function to model this population growth.
15. Without graphing, determine whether each of the following functions represents exponential growth, exponential decay, or neither.
a. $y=129(1.63)^{x} 6$
b. $y=2(.065)^{x}$
c. $y=12\left(\frac{12}{7}\right)^{x}$
6
d. $y=1^{x}$
$N$
e. $y=\frac{2}{37}\left(\frac{4}{5}\right)^{x} \quad D$
f. $y=7\left(\frac{x}{7}\right)^{2} \quad$ U

Graph the function. Identify the horizontal asymptote, domain, and range.
16.
$y=(0.75)^{x}$


Asymptote: $y=0$
Domain: $\qquad$ Range: $y>0$
17.
$y=8(0.5)^{x+2}+3$


Asymptote: $y=0$
Domain: $\qquad$ Range: $y>3$
18. Suppose you invest $\$ 2000$ at an annual rate of $4.5 \%$ compounded quarterly.
a. How much will you have in your account after 5 years? $\$ 2501.50$
b. How much would you have in your account if it compounds continuously instead?

$$
\$ 2504.65
$$

19. A machine shop purchases a piece of equipment for $\$ 300,000$. The value of the equipment depreciates at a rate of $14 \%$ each year.
a. Write the decay model for the value of the equipment.

$$
y=300,000(.86)^{x}
$$

b. What is the value of the equipment after the sixth year?

$$
121,370.17
$$

c. Use the model to estimate when the equipment will have a value of $\$ 50,000$.

$$
11.88 \text { yrs }
$$

20. A house was purchased for $\$ 154,000$ in 1995 . The value of the home increases by $3 \%$ each year.
a. Write the growth model for the value of the house

$$
y=154,000(1.03)^{x}
$$

b. What is the value of the house after the tenth year?

$$
\$ 206,963.12
$$

c. Use the model to estimate the year when the house will have a value of $\$ 500,000.39 .8 \mathrm{y} \mathrm{rs}$.

Solve each exponential equation.
21. $3^{4}=27^{x-1}$
22. $4^{5 x}=8^{6-x}$
23. $\left(\frac{1}{125}\right)^{2 x-3}=5^{5 x-2}$
24. $9^{5 x}=27^{2 x-1}$

$$
x=7 / 3
$$


26. $5^{x}=21$
27. $8^{2 x}=18$
28. $3^{x+1}=17$
$x \approx 1.892$
$x \approx .695$ $x \approx 1.579$
29. $4^{5 x-3}+9=14$
30. $8-3^{x}=-1$


$$
x=2
$$

Solve each exponential word problem.
31. A new car that sells for $\$ 18,500$ depreciates $22 \%$ each year. Write a function that models the value of the car. Find the value of the car after 14 years

$$
\$ 570.82
$$

32. The population of an endangered bird is increasing at a rate $7.5 \%$ per year. There are currently about 200,000 of these birds. Write a function that models the bird population. How many birds will there be in 23 years? How many birds were there 15 years ago?

$$
\begin{aligned}
y & =200,000(1.075)^{t} \\
f(23) & =1,055,418
\end{aligned} \quad f(-15)=67,593
$$

Write each equation in exponential form.
33. $\log _{56} 1=0$
34. $\log _{12} \frac{1}{144}=-2$
35. $\log _{3} \frac{1}{81}=-4$

$$
56^{\circ}=1
$$

$$
12^{-2}=\frac{1}{144}
$$

$$
3^{-4}=\frac{1}{81}
$$

36. $\log _{3} 6561=8$
37. $\log 10=1$
38. $\log _{8} 64=2$

$$
3^{8}=6561
$$

$$
10^{\prime}=10
$$

$$
8^{2}=64
$$

Write each equation in logarithmic form.
39. $3^{2}=9$
40. $4^{-2}=\frac{1}{16}$

$$
\log _{3} 9=2
$$

$$
\log _{4} \frac{1}{16}=-2
$$

42. $2^{-4}=\frac{1}{16}$
43. $8^{3}=512$
44. $17^{1}=17$

$$
\log _{17} 17=1
$$

$$
\log _{2} \frac{1}{16}=-4
$$

$$
\log _{8} 512=3
$$

44. $4^{-3}=\frac{1}{64}$

$$
\log _{4} \frac{1}{64}=-3
$$

Expand each logarithmic expression.
45. $\log 3 x^{5}$
46. $\log _{5} 6 \sqrt{7 y}$
$\log 3+5 \log x$

$$
\log _{5} 6+\frac{1}{2} \log _{5} 7+\frac{1}{2} \log _{5} y
$$

47. $\log \frac{y^{2}}{4 x}$

$$
2 \log y-\log 4-\log x
$$

48. $\log _{2} \frac{x^{9} y^{5}}{z^{2}}$
49. $\log _{3} \sqrt{3 y z}$
50. $\log _{3} 2 x^{-4} y^{9}$
$9 \log _{2} x+5 \log _{2} y-2 \log _{2} 7$

$$
\frac{1}{2} \log _{3} 3+\frac{1}{2} \log _{3} y+\frac{1}{2} \log _{3} 2
$$

$$
\log _{3} 2-4 \log _{3} x+9 \log _{3} y
$$

Write each expression as a single log.
51. $\log _{5} 20-\log _{5} 12$

$$
\log _{5}\left(\frac{5}{3}\right)
$$

54. $7 \log _{4} 2+\log _{4} x+3 \log _{4} y$

$$
\log _{4} 128 x y^{3}
$$

52. $2 \log 5-4 \log x^{3}$

$$
\log \left(\frac{25}{x^{2}}\right)
$$

55. $\log _{3} 4+2 \log _{3} x-\log _{3} 5$

$$
\log _{3} \frac{4 x^{2}}{5}
$$

53. $3 \ln x+\ln 9$

$$
\ln 9 x^{3}
$$

56. $\log 9-\log 3-\log 5$

$$
\log \frac{3}{5}
$$

59. $\log _{2} 64=8 x$

$$
x=3 / 4
$$

60. $\log _{x} 125=3$
61. $\log _{8}(m+1)=\log _{8} 2 m$
62. $\log 5 x=2$

$$
m=1 \quad x=20
$$

Find the inverse of each function.
63. $y=\ln 5 x$

$$
y=\frac{e^{x}}{5}
$$

64. $y=\log (x+3)$

$$
y=10^{x}-3
$$

65. $y=\log _{5}(3 x+2)$

$$
y=\frac{5^{x}-2}{3}
$$

Simplify.
66. $\log _{10} 10^{5}=5$

Identify the parent function of each equation and describe how each of the following would translate from the parent function.
67. $y=\log _{5}(x)+3$
68. $\log _{3}(x-2)-6$
69. $\quad \ln (x+1)+4$
up 3
$R 2$, down b
$L 1$, up 4
70. $y=\left(\frac{1}{2}\right)^{x+1}+3$
71. $y=8(6)^{x-4}-2$
72. $y=2\left(\frac{3}{5}\right)^{x-3}+8$

$$
L 1, \text { up } 3
$$

$R 4$, down 2
$R 3$, up 8

Determine if each of the functions below are examples of exponential growth or exponential decay.
73. $f(x)=5 \cdot 2^{x}-6$
74. $f(x)=\frac{1}{3} \cdot\left(\frac{9}{4}\right)^{x-2}+2$
6
75. $f(x)=2(0.4)^{x}$
76. $f(x)=\left(\frac{2}{5}\right)^{x}+1 \quad D$
77. $y=250(.75)^{t}$
a) Growth or decay?
b) What is the growth/decay factor? .75
b) What is the growth/ decay factor? 1.05
c) What is the percent increase/ decrease? $25 \%$
c) What is the percent increase/decrease? $5 \%$
d) What is the initial amount? 250

Match the following logarithmic equations to their graphs below. Then identify the domain, range and asymptote of each graph.

| $\log (x)$ | $\log _{3}(x+1)$ | $\log (x)+2$ | $\log (x+2)$ |
| :--- | :--- | :--- | :--- |
| $\log _{3}(x-1)+2$ | $\log _{3}(x-1)$ | $\log (x-3)$ | $\log _{3}(x)-3$ |

\#1
EQ: $\log x$


Domain: $\qquad$ $x>0$
Range: $\qquad$
Asymptote: $X=0$


Domain: $x>-2$
Range: $\qquad$
Asymptote: $X=-2$



Domain: $X>0$
Range: $\qquad$
Asymptote: $X=0$
${ }^{45}$ Ea: $\log _{3}(x+1)$


Domain: $x>-1$
Range: $\qquad$
Asymptote: $x=-1$


Domain: $\qquad$
Range: $\qquad$
Asymptote: $X=0$

$$
{ }_{\text {Ea b }}^{46}: \log _{3}(x-1)+2
$$



Domain: $\qquad$ $x>1$
Range: $\qquad$
Asymptote: $X=1$

## Compound Interest Problems

79. If $\$ 9100$ is deposited in an account at the bank and it ears $12 \%$ annual interest, compounded continuously, what is the amount in the account after 7 years?

$$
\$ 21,078.94
$$

80. If $\$ 3900$ is deposited in an account at the bank and it earns $9 \%$ annual interest, compounded continuously, what is the amount in the account after 10 years?

$$
\$ 9.592 .45
$$

81. Find the value of $\$ 1000$ deposited for 10 years in an account paying $7 \%$ annual interest compounded yearly.

$$
\$ 1,967.15
$$

82. Find the value of $\$ 1000$ deposited for 8 years in an account paying $8 \%$ annual interest compounded semiannually.

$$
\$ 1,872.98
$$

83. An investment of $\$ 7,400$ at $12 \%$ per year is compounded quarterly. How much will the investment be worth in 15 years?

## $\$ 43,597.86$

| $n$th term of a geometric <br> sequence | Sum of a geometric <br> series |
| :---: | :---: |
| $a_{n}=a_{1} \cdot r^{n-1}$ | $S_{n}=\frac{a_{1}\left(1-r^{n}\right)}{(1-r)}$ |
| $a_{n}=\left\{\begin{array}{l}a_{1}, n=1 \\ r \cdot a_{n-1}, n>1\end{array}\right.$ |  |

Determine the type of sequence (Arithmetic or Geometric or Neither).
84. $15,9,3,-3,-9,-15$

Arith
85. $64,16,4,1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}$

Geo
86. $1,1,2,6,24,120, \ldots$

87. 200, 20, 2, 0.2, 0.02,...

Find the indicated term of each Geometric Sequence.
88. $a_{1}=5$

89. $a_{1}=2 \quad a_{2}=-12$
$a_{5}=2,592$
90.

$$
a_{1}=-4 \quad r=-2 \quad a_{6}=128
$$

91. $a_{3}=8 \quad a_{5}=2$ $\qquad$

## Write the series with summation notation.

92. $6+12+18+\ldots+96$

$$
\text { Ship }=\text { Arithmetic }
$$

93. Find the first 4 terms of the geometric sequence for which $a_{1}=8$ and $r=5$.

$$
8,40,200,1000
$$

94. Identify the next three terms in the sequence. $3,6,12,24, \ldots$

$$
48,96,192
$$

95. Find the missing number in the following pattern.

$$
2,1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8},-\frac{1}{16}
$$

## Evaluate the following Sigma Notation.

96. $\sum_{n=1}^{5} \frac{2}{3}(2)^{n}$

97. Find the common ratio of the sequence.

98. Write an explicit and recursive rule for the $n$th term of the geometric sequence. $a_{1}=5$ and $r=-\frac{1}{2}$

$$
a_{n}=5\left(-\frac{1}{2}\right)^{n-1} \quad a_{n}=\left\{\begin{array}{c}
5, n=1 \\
a_{n-1}\left(-\frac{1}{2}\right), n>1
\end{array}\right.
$$

99. Find the sum of the geometric series: $30-6+\frac{6}{5}-\frac{6}{25}+\ldots$ Find sum of 1 st 5 terms.

$$
\frac{521}{625}
$$

100. Write a recursive formula for the sequence.

$$
-500,100,-20,4, \ldots
$$

[A] $a_{1}=-500 ; a_{n}=-\frac{1}{5}\left(a_{n-1}\right)$
[B] $a_{1}=-500 ; a_{n}=-5\left(a_{n-1}\right)$
[C] $a_{1}=4 ; a_{n}=-5\left(a_{n-1}\right)$
[D] $a_{1}=4 ; a_{n}=-\frac{1}{5}\left(a_{n-1}\right)$

The box plot below shows the dollar prices of twenty popcorn poppers as listed in Consumer Reports Buying Guide, 1981.


Source: Consumer Reports Buying Guide, 1981.
101. Answer the following questions based on the information given in the graph
a. Approximately how much did the most expensive popcorn popper cost? $\$ 48$
b. Approximately how much did the least expensive popcorn popper cost?
c. What was the median price for a popcorn popper? \$21
d. What percentage of the poppers cost more than $\$ 26.50$ (the upper quartile)? $25 \%$
e. What percentage of the poppers cost more than $\$ 17.00$ (the lower quartile)? $75 \%$
f. What is the range of prices? 36
g. What is the interquartile range of prices? 9.50
102. The histograms below represent average weekly job income for students in two high school classes.



What conclusion is reasonable based on the displays? (Note: Both classes have medians in the $\$ 200$ range.)
(IAD The mean weekly income for class $A$ is higher than for class $B$.
[B] The mean weekly income for class $B$ is higher than for class $A$.
[C] More students in class $B$ have higher paying jobs than in class $A$.
[D] All students in both classes have paying jobs
103. What are the measures of central tendency?
Mean, median, mode
104. If a student has the following test scores: $84 \%, 92 \%, 85 \%, 88 \%$, and $91 \%$
a. Find the mean
72
b. Find the median 85
c. Find the range 8
d. If the next test score is $67 \%$, what measure would be most affected the mean or the range? Explain.
Range
105. What does the standard deviation tell about a set of data? Distance from mean
106. Which data set would give the smallest standard deviation?
a. $\{100,200,300,400\}$
(b) $\{100,101,102,103\}$
107. Find the 5 number summary and draw a box-and-whisker plot. Round your answers to the nearest tenth.

$$
8,11,15,28,32
$$

$10,11,15,8,26,30,28,20,32,15,12$

108. Find the margin of sampling error for a survey with the given sample size. Round your answer to the nearest tenth of a percent.
a) 1,000
$3.2 \%$
b) 2500
$2 \%$
c) $3675.2 \%$
d) $350 \quad 5.3 \%$
109. Find the smallest sample size required for the given margin of sampling error. Round your answer to the nearest whole number.
a. $4.8 \%$
b. $2.3 \%$
c. $7.4 \%$
434
1890
183
110. A random sample of 2500 consumers, reports that $61 \%$ prefer game $A$ over game $B$. Which interval best represents the actual percent of people in the population likely to prefer game $A$ over game $B$ ?
A. $60.5 \%-61.5 \%$
B. $60 \%-62 \%$
C. $59 \%-63 \%$
D. 56\%-66\%

In exercises 111-113 use the information below.

In a survey of 802 people, $16 \%$ said that they use the internet or e-mail more than 10 hours per week.
111. What is the margin of error for the survey? Round your answer to the nearest tenth.

$$
3.5 \%
$$

112. Give an interval that is likely to contain the exact percent of all people who use the internet or e-mail more than 10 hours per week.

$$
12.5 \%-19.5 \%
$$

113. Explain how the margin of error for the survey can be made smaller.
langer sample size
114. The data shows the grades of students in a test. Which of the following graphs represents the data on a histogram? $\begin{array}{lllllllllllllllllllllllllllll}50 & 51 & 52 & 53 & 54 & 71 & 73 & 72 & 74 & 66 & 67 & 66 & 68 & 61 & 62 & 63 & 64 & 69 & 68 & 67 & 66 & 56 & 58 & 59 & 57 & 59 & 56\end{array}$

115. ACT scores are normally distributed with a mean of 18 and a standard deviation of 6. Label a normal curve with the values and then answer the questions.
a) What percent get at least an 18 ? $50 \%$
b) What percent get at least a 30 ? $2.5 \%$
c) What percent get at most 12 ? $16 \%$
d) Between what scores does $68 \%$ of the data fall? $12-24$
116. A normal distribution has a mean of 20 and a standard deviation of 2.

7 Find the given probability.
a. Three randomly selected x -valued are all 26 or greater

## .000000003

b. Two randomly selected $x$-values are both 16 or less

$$
.000625
$$

## Part IV - Probability (Topic 12)

117. Each event can occur in the given number of ways. Find the number of ways all of the events can occur.

Event A: 5 ways, Event B: 7 ways

## 35

118. A lunch menu consists of 5 different sandwiches, 4 different soups, and 4 different drinks. How many choices are there for ordering a sandwich, a bowl of soup, and a drink?

## 80

1) 2 . Find the number of distinguishable permutations of the letters GRAPHICS.
[A] 40,320
[B] 28
[C] 64
[D] 56
stip
12. Find the number of distinguishable permutations of the letters SWEET. sfip
13. You own 5 pairs of shoes and are taking 2 of them on vacation. In how many ways can you choose 2 pairs of shoes from the 5 ?
14. A college has ten instructors qualified to teach a special computer lab course which requires two instructors to be present. How many different pairs of teachers could there be?
[A] 90
[B] 36
[C] 20
[D] 45
15. From a group of five boys and six girls, a boy and a girl will be selected to attend a conference. In how many possible ways can the selection be made?
16. A four-person committee is chosen at random from a group of 15 people. How many different committees are possible?

## 1365

125. How many different 3 -card hands can be drawn from a standard deck of 52 playing cards?

$$
22,100
$$

126. A card is drawn from a standard deck of playing cards.

Find the probability that it is NOT a face card ( $\mathbf{J}, \mathbf{Q}$, or $K$ ).

127. A card is drawn from a standard deck of playing cards. Find the probability that it is an ace or a heart.

128. A number cube with faces numbered $1,2,3,4,5$, and 6 is rolled. Find the probability of rolling a number greater than 4.

$$
\frac{1}{3}
$$

129. Three cards are randomly selected from a 52 card deck without replacement. What is the probability of getting 3 spades?
[A] 0.012
[B] 0.013
[C] 0.015
[D] 0.016
130. A six-sided die is rolled 450 times. Five comes up 77 times.
a. What is the theoretical probability of rolling a five?

b. What is the experimental probability of rolling a five?

$$
\frac{77}{450}
$$

131. A number cube is rolled 430 times and the results recorded as follows: there were 68 ones, 73 twos, 62 threes, 71 fours, 70 fives, and 86 sixes. What is the experimental probability of rolling a number greater than four?
[A] 0.54
[B] 0.64
[C] 0.36
[D] 0.46
132. Refer to the spinner shown.

The spinner is divided into sections with the same area.
a. What is the probability that the spinner stops on a multiple of 3 ?

$$
\frac{9}{10}
$$

b. What are the odds in favor of stopping on a multiple of 4 ?


## Ship

c. What are the odds against stopping on a number less than 12 ?
ship
133. Find the probability that a randomly thrown dart will hit the shaded region.

134. Eight balls numbered from 1 to 8 are placed in an urn. Two balls are selected at random without replacement. Find the probability of drawing ball \#3 then ball \# 5

$$
\frac{1}{56}
$$

135. $A$ and $B$ are two events. $P(A)=0.54 ; P(B)=0.42 ; P(A$ or $B)=0.85$. Find the probability of $A$ and $B$.
136. Eight balls numbered from 1 to 8 are placed in an urn. One ball is selected at random. Find the probability that it is NOT number 4.

137. A drawer contains 2 red socks, 6 white socks, and 4 blue socks. Without looking, you draw out a sock, return it, and draw out a second sock. What is the probability that the first sock is blue and the second sock is red?

138. A coin is tossed and a die is rolled. What is the probability that the coin shows heads and the die shows a 3 or a 4 ?
[A] $\frac{1}{12}$
[B] $\frac{1}{6}$
[C] $\frac{5}{6}$
[D] $\frac{1}{4}$
139. Is following independent?

You spin a spinner twice.
yes
140. Is the following independent?

You draw a red card from a deck with an equal number of red and blue cards and without replacing it, draw another red card. ho
141. Which events are independent?
[A] You choose 2 different ice cream flavors.
[B] You draw 2 colored cards at the same time and get one red and one green.
[C] You complete the review packet and pass the final.
[D] You draw a card from a deck, replace it and draw a second.
142. $A$ and $B$ are independent events.

$$
P(A)=0.6 \quad P(B)=0.8 \quad \text { Find } P(A \text { and } B) \text {. }
$$

$$
.48
$$

143. A jar contains 21 green marbles and 30 yellow marbles. One marble is withdrawn and the color noted. It is then returned to the jar, mixed in, and another marble is withdrawn. Find the probability that both marbles are green.

$$
\frac{49}{289}
$$

144. If $P(A)=0.72$, what is If $P\left(A^{\prime}\right)$ ?

$$
.28
$$

145. The probability for snow today is 0.4 . The probability for snow tomorrow is 0.35 . The probability for snow both days is 0.17 .
a. Find the probability that it will snow today or tomorrow.

$$
.58
$$

W. Find the probability that it will snow at least one of the two days.
146. Write in standard form the equation of the circle with center $(2,3)$ and radius 4.

$$
(x-2)^{2}+(y-3)^{2}=16
$$

147. Find the center and radius of the circle $x^{2}+y^{2}+4 x-12 y-2=0$.

$$
C=(-2,6) \quad r=6.48
$$

148. Write in standard form the equation of the parabola with focus $(0,2)$ and directrix $y=-4$. Use the graph as an aid if you need it.


$$
x^{2}=12(y+1)
$$

149. Find the vertex, axis of symmetry, focus, and the directrix of the parabola $y=\frac{1}{8}(x-1)^{2}-4$. Use the graph as an aid if you need it.


Vertex: $(1,-4)$ Focus: $(1,-2)$ Directrix: $y=-6$ Axis: $x=1$
150. Find the vertex, focus, axis of symmetry and directrix of the parabola $x=\frac{1}{4}(y-2)^{2}+4$. Use the graph as an aid if you need it.


