5.4-5.5 Review
AA2

Name: Key
Hr: ____

Solve. Remember to check logarithmic equations for extraneous solutions. Check with the exact answers. The approximate answers are listed in the key.

1.) $6^x = \frac{1}{216}$

$6^x = 6^{-3}$

$\boxed{x = -3}$

2.) $3(5^{x-1}) - 7 = 28$

$3(5^{x-1}) = 35$

$5^{x-1} = \frac{35}{3}$

$\log_5 5^{x-1} = \log_5\left(\frac{35}{3}\right)$

$\boxed{x = \log_5\left(\frac{35}{3}\right) + 1 \approx 2.5}$

3.) $4^{(x-4)} \cdot 8^x = 4^{(x+4)}$

$2^{2(x-4)} \cdot 2^{3x} = 2^{2(x+4)}$

$2x - 8 + 3x = 2x + 8$

$5x = 2x + 16$

$3x = 16$

$\boxed{x = 16/3}$

4.) $\frac{2^{4x+12}}{2^{2x}} = 2^{3x-9}$

$4x + 12 - (2x) = 3x - 9$

$2x + 12 = 3x - 9$

$12 = x - 9$

$\boxed{21 = x}$

5.) $e^x = 3$

$\ln e^x = \ln 3$

$\boxed{x = \ln 3 \quad x \approx 1.099}$

6.) $e^{4x} = e^{x^2+3}$

$4x = x^2 + 3$

$0 = x^2 - 4x + 3$

$0 = (x-3)(x-1)$

$\boxed{x = 3,\ x = 1}$

7.) $e^{2x} - 7e^x + 10 = 0$

$(e^x - 5)(e^x - 2) = 0$

$e^x = 5 \qquad e^x = 2$

$\boxed{x = \ln 5 \quad x = \ln 2 \quad x \approx 1.609 \quad x \approx .693}$

8.) $\left(1 + \frac{0.10}{12}\right)^{12t} = 2$

$(1.008)^{12t} = 2$

$\log_{1.008} 12t = \log_{1.008} 2$

$12t = \log_{1.008} 2$

$\boxed{t = \frac{\log_{1.008} 2}{12} \approx 6.960}$

9.) $\log_4 x = 2$

$4^2 = x$

$\boxed{16 = x}$

10.) $2\ln 4x = 15$

$\ln 4x = \frac{15}{2}$

$e^{15/2} = 4x$

$\boxed{\frac{e^{15/2}}{4} = x \quad 452.011 \approx x}$

11.) $\ln\sqrt{x+1} = 2$

$e^2 = \sqrt{x+1}$

$e^4 = x + 1$

$\boxed{e^4 - 1 = x \quad 53.598 \approx x}$

12.) $\log_6(x+4) - \log_6 x = \log_6(x+2)$

$\log_6\left(\frac{x+4}{x}\right) = \log_6(x+2)$

$\frac{x+4}{x} = \frac{x+2}{1}$

$x + 4 = x^2 + 2x$

$0 = x^2 + x - 4$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-4)}}{2(1)}$

$x = \frac{-1 \pm \sqrt{17}}{2}$

$\boxed{x \approx 1.562}$

$x \approx \cancel{-2.562}$

13.) On the Richter scale the magnitude R of an earthquake with intensity I is $R = \log\frac{I}{I_0}, I_0 = 1$. Find the intensity when the magnitude is 7.2

$7.2 = \log \frac{I}{1}$

$10^{7.2} = I$

$I = 15{,}848{,}931$

14.) You deposit money in a savings account. Find the amount of time to triple your money if it is compounded continuously at 3.5%.

$3 = 1e^{.035t}$

$\ln 3 = .035t \cancel{\ln e}$

$\frac{\ln 3}{.035} = t$

$t \approx 31.34$ YEARS

15.) In a typing class the average number N of words per minute typed after t weeks of lessons was $N = \frac{157}{1+5.4e^{-0.12t}}$. Find the time necessary to type 75 words per minute.

$75 = \frac{157}{1+5.4e^{-0.12t}}$

$75(1+5.4e^{-0.12t}) = 157$

$1+5.4e^{-0.12t} = 157/75$

$5.4e^{-0.12t} = \frac{157}{75} - 1$

$e^{-0.12t} = \left[\frac{157}{75} - 1\right] / 5.4$

$-0.12t = \ln\left[\frac{\frac{157}{75} - 1}{5.4}\right]$

$t = \frac{\ln\left[\frac{\frac{157}{75}-1}{5.4}\right]}{-0.12} \approx 13.3$

16.) The half-life of ^{239}Pu is 24,100 years. Find the initial amount if the amount after 1000 years is 0.4 grams.

$y = ae^{kt}$

$1 = 2e^{k(24,100)}$

$.5 = e^{24,100k}$

$\ln(.5) = 24,100k$

$k = \frac{\ln(.5)}{24,100}$

$0.4 = ae^{\ln .5/24,100(1000)}$

$\frac{0.4}{e^{\ln .5/24,100(1000)}} = a$

$\boxed{a \approx .412g}$

17.) The pH is a measure of the hydrogen ion concentration [H+] of a solution. The model is ph = -log[H+]. Find [H+] if the pH is 3.2.

$3.2 = -\log[H+]$

$-3.2 = \log[H+]$

$10^{-3.2} = [H+]$

$\boxed{[H+] \approx 6.31 \times 10^{-4}}$

18.) You deposit money into an account that is compounded continuously at 2%. After 10 years the amount is \$2000. Find the initial investment AND the time to double your deposit.

$A = Pe^{rt}$

$2000 = Pe^{.02(10)}$

$2000 = Pe^{.2}$

$\frac{2000}{e^{0.2}} = P$

$P = \$1637.46$

$2 = 1e^{.02t}$

$\ln 2 = .02t$

$\frac{\ln 2}{.02} = t$

$t \approx 34.657$ YEARS

KEY:

1.) -3 2.) ≈2.5 3.) 16/3 4.) 21 5.) ≈1.099 6.) 1, 3 7.) ≈0.693, 1.609 8.) ≈6.960 9.) 16 10.) ≈452.011 11.) ≈53.598
12.) ≈1.562 13.) 15,848,931 14.) ≈31.4 15.) ≈13.3 16.) ≈0.41 17.) 6.3 x 10^{-4} 18.) \$1637.46 and 34.7 yrs