5.4-5.5 BOOK REVIEW

$\overline{x-1} - \ln(x+1)^2$

$\frac{\overline{x-1}}{+1)^2}$

$\ln(x-2)^5 - \ln(x+2) - \ln x^3$

$= \ln(x-2)^5 - [\ln(x+2) + \ln x^3]$

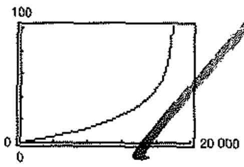
$= \ln(x-2)^5 - \ln x^3(x+2)$

$= \ln \frac{(x-2)^5}{x^3(x+2)}$

. $t = 50 \log \frac{18{,}000}{18{,}000 - h}$

(a) Domain: $0 \le h < 18{,}000$

(b)



Vertical asymptote. $h = 18{,}000$

(c) As the plane approaches its absolute ceiling, it climbs at a slower rate, so the time required increases

(d) $50 \log \frac{18{,}000}{18{,}000 - 4000} \approx 5.46$ minutes

96. Using a calculator gives $s = 84.66 + (-11 \ln t)$.

97. $8^x = 512$

$8^x = 8^3$

$x = 3$

98. $6^x = \frac{1}{216}$

$6^x = 6^{-3}$

$x = -3$

99. $e^x = 3$

$x = \ln 3$

100. $e^x = 6$

$\ln e^x = \ln 6$

$x = \ln 6 \approx 1.792$

101. $\log_4 x = 2$

$x = 4^2 = 16$

102. $\log_6 x = -1$

$6^{\log_6 x} = 6^{-1}$

$x = \frac{1}{6}$

103. $\ln x = 4$

$x = e^4$

104. $\ln x = -3$

$x = e^{-3} \approx 0.0498$

105. $e^x = 12$

$\ln e^x = \ln 12$

$x = \ln 12 \approx 2.485$

106. $e^{3x} = 25$

$\ln e^{3x} = \ln 25$

$3x = \ln 25$

$x = \frac{\ln 25}{3} \approx 1.073$

107. $e^{4x} = e^{x^2+3}$

$4x = x^2 + 3$

$0 = x^2 - 4x + 3$

$0 = (x-1)(x-3)$

$x = 1$ or $x = 3$

108. $14e^{3x+2} = 560$

$e^{3x+2} = 40$

$\ln e^{3x+2} = \ln 40$

$3x + 2 = \ln 40$

$x = \frac{(\ln 40) - 2}{3} \approx 0.563$

109. $2^x + 13 = 35$

$2^x = 22$

$x = \log_2 22$

$= \frac{\log 22}{\log 2}$ or $\frac{\ln 22}{\ln 2}$

$x \approx 4.459$

110. $6^x - 28 = -8$

$6^x = 20$

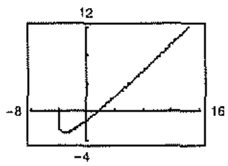
$\log_6 6^x = \log_6 20$

$x = \log_6 20$

$x = \frac{\ln 20}{\ln 6} \approx 1.672$

134. $x - 2\log(x + 4) = 0$

Let $y_1 = x - 2\log(x + 4)$



The x-intercepts are at $x \approx -3.990$ and $x \approx 1\ 477$

135. $3(7550) = 7550e^{0\ 0725t}$

$$3 = e^{0\ 0725t}$$

$$\ln 3 = \ln e^{0\ 0725t}$$

$$\ln 3 = 0\ 0725t$$

$$t = \frac{\ln 3}{0\ 0725} \approx 15\ 2 \text{ years}$$

136. $S = 93\log(d) + 65$

$$283 = 93\log(d) + 65$$

$$218 = 93\log(d)$$

$$\log(d) = \frac{218}{93}$$

$$d = 10^{(218/93)} \approx 220\ 8 \text{ miles}$$

137. $y = 3e^{-2x/3}$

Exponential decay model

Matches graph (e)

138. $y = 4e^{2x/3}$

Exponential growth model

Matches graph (b).

139. $y = \ln(x + 3)$

Logarithmic model

Vertical asymptote: $x = -3$

Graph includes $(-2, 0)$

Matches graph (f)

140. $y = 7 - \log(x + 3)$

Logarithmic model

Vertical asymptote $x = -3$

Matches graph (d)

141. $y = 2e^{-(x+4)^2/3}$

Gaussian model

Matches graph (a)

142. $y = \dfrac{6}{1 + 2e^{-2x}}$

Logistics growth model

Matches graph (c)

143. $y = ae^{bx}$

$$2 = ae^{b(0)} \implies a = 2$$

$$3 = 2e^{b(4)}$$

$$1\ 5 = e^{4b}$$

$$\ln 1\ 5 = 4b \implies b \approx 0\ 1014$$

Thus, $y \approx 2e^{0\ 1014x}$

144. $y = ae^{bx}$

$$\frac{1}{2} = ae^{b(0)} \implies a = \frac{1}{2}$$

$$5 = \frac{1}{2}e^{b(5)}$$

$$10 = e^{5b}$$

$$\ln 10 = 5b$$

$$\frac{\ln 10}{5} = b$$

$$b \approx 0\ 4605$$

$$y = \frac{1}{2}e^{0\ 4605x}$$

5. $P = 3499e^{0.0135t}$

$4.5 \text{ million} = 4500 \text{ thousand}$

$4500 = 3499e^{0.0135t}$

$\frac{4500}{3499} = e^{0.0135t}$

$\ln\left(\frac{4500}{3499}\right) = 0.0135t$

$t = \frac{\ln(4500/3499)}{0.0135} \approx 18.6 \text{ years}$

According to this model, the population of South Carolina will reach 4.5 million during the year 2008.

146. $y = Ce^{kt}$

$\frac{1}{2}C = Ce^{(250,000)k}$

$\ln\frac{1}{2} = \ln e^{(250,000)k}$

$\ln\frac{1}{2} = 250,000k$

$k = \frac{\ln(1/2)}{250,000}$

When $t = 5000$, we have

$y = Ce^{[\ln(1/2)/250,000](5000)} \approx 0.986C = 98.6\%C$

After 5000 years, approximately 98.6% of the radioactive uranium II will remain.

147. (a) $20,000 = 10,000e^{r(5)}$

$2 = e^{5r}$

$\ln 2 = 5r$

$\frac{\ln 2}{5} = r$

$r \approx 0.138629$

$= 13.8629\%$

(b) $A = 10,000e^{0.138629}$

$\approx \$11,486.98$

148. $N_0 = 2000$ and $N_3 = 1400$ so $N = 2000e^{kt}$ and.

$1400 = 2000e^{3k}$

$\frac{7}{10} = e^{3k}$

$3k = \ln\left(\frac{7}{10}\right)$

$k = \frac{\ln(7/10)}{3} = -0.11889$

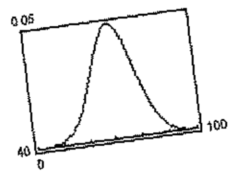
The population one year ago:

$N(4) = 2000e^{-0.11889(4)}$

$= 1243 \text{ bats}$

149. $y = 0.0499e^{-(x-71)^2/128}$, $40 \le x \le 100$

(a) Graph $y_1 = 0.0499e^{-(x-71)^2/128}$.



(b) The average test score is 71.

150. $N = \frac{157}{1 + 5.4e^{-0.12t}}$

(a) When $N = 50$:

$50 = \frac{157}{1 + 5.4e^{-0.12t}}$

$1 + 5.4e^{-0.12t} = \frac{157}{50}$

$5.4e^{-0.12t} = \frac{107}{50}$

$e^{-0.12t} = \frac{107}{270}$

$-0.12t = \ln\frac{107}{270}$

$t = \frac{\ln(107/270)}{-0.12} \approx 7.7 \text{ weeks}$

(b) When $N = 75$:

$75 = \frac{157}{1 + 5.4e^{-0.12t}}$

$1 + 5.4e^{-0.12t} = \frac{157}{75}$

$5.4e^{-0.12t} = \frac{82}{75}$

$e^{-0.12t} = \frac{82}{405}$

$-0.12t = \ln\frac{82}{405}$

$t = \frac{\ln(82/405)}{-0.12} \approx 13.3 \text{ weeks}$

151.
$$\beta = 10\log\left(\frac{I}{10^{-16}}\right)$$
$$125 = 10\log\left(\frac{I}{10^{-16}}\right)$$
$$12.5 = \log\left(\frac{I}{10^{-16}}\right)$$
$$10^{12.5} = \frac{I}{10^{-16}}$$
$$I = 10^{-3.5} \text{ watt/cm}^2$$

152. $R = \log I$ since $I_0 = 1$.

(a) $\log I = 8.4$

$I = 10^{8.4} \approx 251{,}188{,}643$

(b) $\log I = 6.85$

$I = 10^{6.85} \approx 7{,}079{,}458$

(c) $\log I = 9.1$

$I = 10^{9.1} \approx 1{,}258{,}925{,}412$

153. True. By the inverse properties, $\log_b b^{2x} = 2x$

154. False. $\ln x + \ln y = \ln(xy) \neq \ln(x + y)$

155. Since graphs (b) and (d) represent exponential decay, b and d are negative.
Since graph (a) and (c) represent exponential growth, a and c are positive.

Problem Solving for Chapter 5

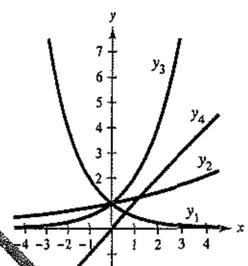
1. $y = a^x$

$y_1 = 0.5^x$

$y_2 = 1.2^x$

$y_3 = 2.0^x$

$y_4 = x$



The curves $y = 0.5^x$ and $y = 1.2^x$ cross the line $y = x$. From checking the graphs it appears that $y = x$ will cross $y = a^x$ for $0 \le a \le 1.44$.

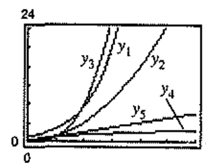
2. $y_1 = e^x$

$y_2 = x^2$

$y_3 = x^3$

$y_4 = \sqrt{x}$

$y_5 = |x|$



The function that increases at the fastest rate for "large" values of x is $y_1 = e^x$. (*Note.* One of the intersection points of $y = e^x$ and $y = x^3$ is approximately (4.536, 93) and past this point $e^x > x^3$. This is not shown on the graph above.)

3. The exponential function, $y = e^x$, increases at a faster rate than the polynomial function $y = x^n$.

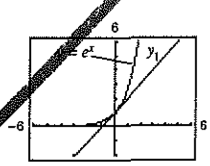
4. It usually implies rapid growth.

5. (a) $f(u + v) = a^{u+v}$
$$= a^u \cdot a^v$$
$$= f(u) \cdot f(v)$$

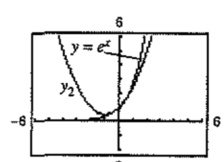
(b) $f(2x) = a^{2x}$
$$= (a^x)^2$$
$$= [f(x)]^2$$

6.
$$[f(x)]^2 - [g(x)]^2 = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$$
$$= \left(\frac{e^{2x} + 2 + e^{-2x}}{4}\right) - \left(\frac{e^{2x} - 2 + e^{-2x}}{4}\right)$$
$$= \frac{4}{4}$$
$$= 1$$

7. (a)



(b)



(c)